Fourier Series：把函數用 sin 和 cos 展開.

$$F[f(x)] = \frac{\boxed{-}\int_T \quad}{} + \frac{\boxed{-}\sum\int_T \quad}{\text{(積一个週期)}} + \frac{\boxed{-}\sum\int_T \quad}{}$$

normalized constant.

$\sin \omega_n x$. $\cos \omega_m x$ 是「良好的」基底 → 具有「正交性」和「完備性」

Show ①正交性 和找 ②normalized constant：

① $\int_0^T \sin mx \sin nx \, dx =$ (T = 2π)

$(m, n = 0, 1, 2 \cdots \text{ \& } m \neq n)$

② $\int_0^T \sin^2 mx \, dx =$

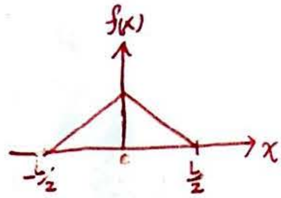
$\int_0^T 1 \, dx =$

HW：積 $\underbrace{\int_0^T \sin mx \cos nx \, dx \, . \, \int_0^T \cos mx \cos nx \, dx}_{\text{show 正交性.}}$. $\underbrace{\int_0^T \cos^2 nx \, dx}_{\text{cos 項的 normalized constant.}}$

做傅立葉展開的流程：

① 判斷函數週期（代入 T）

② 找出"允許的" ω ， ex.



$$\omega = \frac{2\pi}{T} = \frac{2\pi}{L}, \frac{2\pi}{L/2}, \frac{2\pi}{L/3} \cdots$$
$$= \frac{2n\pi}{L} \quad , \quad n = 0, 1, 2 \cdots$$

③ 把找出的 ω 代入，

$$F[f(x)] = \frac{1}{T}\int_T f(x)\,dx + \frac{2}{T}\sum_n \int_T f(x)\sin\omega_n x\,dx \cdot \sin\omega_n x + \frac{2}{T}\sum_n \int_T f(x)\cos\omega_n x\,dx \cdot \cos\omega_n x$$

洪在明老師上課內容 -- 解 wave eqn.

Wave eqn : $\frac{\partial^2}{\partial t^2}u(x,t) = v^2 \frac{\partial^2}{\partial x^2}u(x,t)$

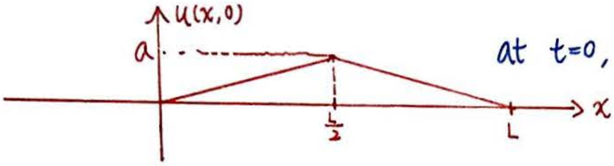
Use separation of variable : Let $u(x,t) =$ ________________

Rewrite wave eqn : ________________ , ________________

Set $\frac{1}{T}\frac{\partial^2 T}{\partial t^2} = \frac{v^2}{X}\frac{\partial^2 X}{\partial x^2} = -\omega^2$ (就是震盪的角頻率)   Note : here $\omega = \frac{2\pi}{T}$, so $\frac{\omega}{k} = v$, $k = \frac{\omega}{v} = \frac{2\pi}{\lambda}$

$\Rightarrow \begin{cases} T = \text{\_\_\_\_\_\_\_\_\_\_\_\_} \\ X = \text{\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$

The wave we want to solve is :



at t=0, release frome rest.

Find "available" $\lambda \Rightarrow \lambda =$ ________________ (so many λ !).

$k = \frac{2\pi}{\lambda} =$ ________________ (many k ...) , $\omega = kv =$ ________________ (many ω ...)

$\Rightarrow \begin{cases} T = \text{\_\_\_\_\_\_\_\_\_\_\_\_} \\ X = \text{\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$

解係數 : 代入 boundary condition ! $\Rightarrow \begin{cases} u(0,0) = 0 \\ u(L,0) = 0 \\ u(x,0) \to \text{fixed.} \\ \dot{u}(x,0) = 0 \quad (\because \text{released frome rest}) \end{cases}$

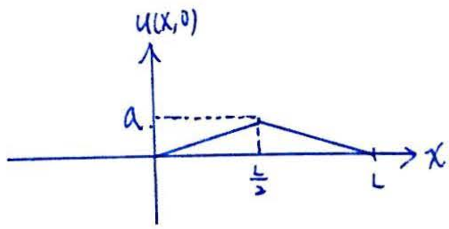
1° $u(0,0) = 0 = u(L,0) \Rightarrow$ ________ $= 0$, $X =$ ________________

2° $\dot{u}(x,0) = 0$, $X \left.\frac{dT}{dt}\right|_{(x,0)} = 0$, $\left.\frac{dT}{dt}\right|_{t=0} = 0 \Rightarrow$ ________ $= 0$, $T =$ ________________

$\Rightarrow u(x,t) = XT =$ ________________

解最後一組係數, use Fourier Series :

At $t=0$, $u(x,0) =$ ________________ =



傅立葉展開 $u(x,0)$!

1° $\because$ $u(x,0) =$ ____________ $\Rightarrow$ u is a _____ function
(even/odd)