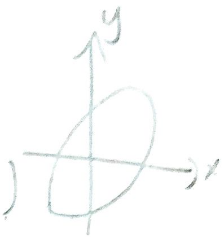
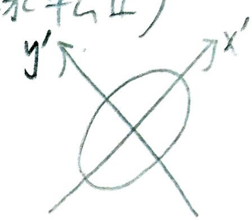
$$(x\ \ y)\begin{pmatrix} a & \frac{b}{2} \\ \frac{b}{2} & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \left(ax+\frac{b}{2}y \quad \frac{b}{2}x+cy\right)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= ax^2+bxy+cy^2$$ (一个斜的二次曲線)



對角化 $\begin{pmatrix} a & \frac{b}{2} \\ \frac{b}{2} & c \end{pmatrix}$: (就是把二次曲線転正)



$$\det\begin{pmatrix} a-\lambda & \frac{b}{2} \\ \frac{b}{2} & c-\lambda \end{pmatrix}=0, \quad (a-\lambda)(c-\lambda)-\frac{b^2}{4}=0,$$

$$\lambda^2+(-a-c)\lambda+\left(ac-\frac{b^2}{4}\right)=0$$

$$\lambda=\frac{a+c\pm\sqrt{(a+c)^2-4ac+b^2}}{2}=\frac{a+c\pm\sqrt{(a+c)^2+D}}{2}$$

$(D=b^2-4ac)$

現在,在$(\hat{x}',\hat{y}')$下的二次曲線:

$$(x'\ \ y')\begin{pmatrix} \frac{a+c+\sqrt{(a+c)^2+D}}{2} & 0 \\ 0 & \frac{a+c-\sqrt{(a+c)^2+D}}{2} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$=\frac{a+c+\sqrt{(a+c)^2+D}}{2}x'^2+\frac{a+c-\sqrt{(a+c)^2+D}}{2}y'^2=Q_1x'^2+Q_2y'^2$$

当 $\begin{cases} D>0 : & Q_1Q_2<0,\ \text{双曲} \\ D=0 : & Q_1Q_2=0,\ \text{抛物} \\ D<0 : & Q_1Q_2>0,\ \text{椭圆} \end{cases}$

證 $tr(U^{-1}AU) = tr(A)$ ，U 是旋転矩陣

旋転矩陣的特性：$U^{-1} = U^T$

$tr(ABC) = \sum_{ijk} A_{ij} B_{jk} C_{ki} = \sum_{ijk} C_{ki} A_{ij} B_{jk} = tr(CAB)$

$tr(U^{-1}AU) = tr(UU^{-1}A) = tr(A)$

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