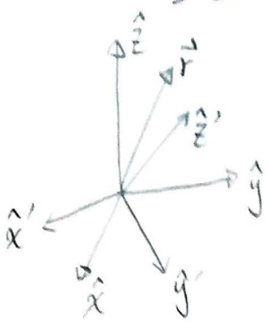
1.1 X

1.2 X

1.3 Coordinate transf.

向量 $\begin{cases} 方向 \\ 量值 \end{cases}$ ⇒ 在某組"基底"下展開



$$\vec{r}_{x,y,z} = (\vec{r}\cdot\hat{x},\ \vec{r}\cdot\hat{y},\ \vec{r}\cdot\hat{z})$$

$$\vec{r}_{x'y'z'} = (\vec{r}\cdot\hat{x}',\ \vec{r}\cdot\hat{y}',\ \vec{r}\cdot\hat{z}')$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \vec{r}\cdot\hat{x}' \\ \vec{r}\cdot\hat{y}' \\ \vec{r}\cdot\hat{z}' \end{pmatrix} = \begin{bmatrix} \hat{x}'_{xyz} \\ \hat{y}'_{xyz} \\ \hat{z}'_{xyz} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \hat{x}'\cdot\hat{x} & \hat{x}'\cdot\hat{y} & \hat{x}'\cdot\hat{z} \\ \hat{y}'\cdot\hat{x} & \hat{y}'\cdot\hat{y} & \hat{y}'\cdot\hat{z} \\ \hat{z}'\cdot\hat{x} & \hat{z}'\cdot\hat{y} & \hat{z}'\cdot\hat{z} \end{bmatrix}}_{\text{Rotation Matrix (転座標軸)}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

1.4. $UU^T = \begin{bmatrix} \hat{x}'\cdot\hat{x} & \cdots \\ \hat{y}' & \cdots \\ \hat{z}' & \cdots \end{bmatrix} \begin{bmatrix} \hat{x}'\cdot\hat{x} & \hat{y}'\cdot\hat{x} & \cdots \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbb{I}$

$\boxed{U^T = U^{-1}}$!

1.5. Matrix Operations

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{2} a_{1j}x_j \\ \sum_{j=1}^{2} a_{2j}x_j \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Write : $\sum_{j=1}^{2} a_{ij}x_j = a_{ij}x_j = u_i$ $\begin{cases} j : \text{dummy index (自动補} \sum_j) \\ i : \text{free index} \end{cases}$

Practice : $A_{ij}B_{jk}C_{kl} = \sum_{?}\sum_{?} A_{ij}B_{jk}C_{kl} \equiv U_{\square\square}$

Ans : $\sum_j \sum_k A_{ij}B_{jk}C_{kl} = U_{il}$.

1.6 A, B, C are Matrices.

$AB \neq BA$

$AA^{-1} = A^{-1}A = \mathbb{1}$

$\mathbb{1}A = A\mathbb{1} = A$

$[AB]C = A[BC]$

$\mathrm{Trace}(U) = U_{ii}$ (求對角線元素和)

$\mathrm{Tr}(ABC) = A_{ij}B_{jk}C_{ki} = \sum_{ijk} A_{ij}B_{jk}C_{ki}$

$\mathrm{Tr}(BCA) = B_{ij}C_{jk}A_{ki} = \sum_{ijk} A_{ki}B_{ij}C_{jk} \overset{\text{change index}}{=} \sum_{ijk} A_{ij}B_{jk}C_{ki}$

$\mathrm{Tr}(CAB) =$ 同上.

$\det(U) = \varepsilon_{ijk} U_{1i} U_{2k} U_{3j}$ (3x3)

$\det(U_{4\times4}) = \varepsilon_{ijkl} U_{1i} U_{2k} U_{3j} U_{4l}$

求 $\det \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $(=0)$.

ε_{ijk} : Levi-Civita Symbol

$\varepsilon_{123} = 1$ (複數次 index 交換)

$\varepsilon_{132} = -1$ (奇 〃 〃 〃 〃)

$\varepsilon_{312} = 1$ ...

其它: ε_{111}、ε_{211}、... $= 0$.

反矩陣.

$$\left[\begin{array}{c|ccc} U & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & U^{-1} \\ 0 & 0 & 1 & \end{array}\right]$$

1.10 Scalar Product

$\vec{A}\cdot\vec{B} = \sum_i A_i B_i = A_i B_i$

1.12 Vector Product

$(\vec{A}\times\vec{B})_i = \sum_{j,k} \varepsilon_{ijk} A_j B_k = \varepsilon_{ijk} A_j B_k$

eqn 1.78

$\sum \varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

Practice $\vec{A}\cdot(\vec{B}\times\vec{D}) \overset{?}{=} \vec{D}\cdot(\vec{A}\times\vec{B})$

$A_i \varepsilon_{ijk} B_j D_k \overset{?}{=} D_i \varepsilon_{ijk} A_j B_k$

(V)

eqn

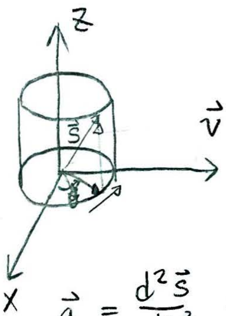
1.81 $A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$

1.82 $A\times B\times C = (A\cdot C)B - (A\cdot B)C$

$\{\varepsilon_{abi}A_b\varepsilon_{ijk}B_jC_k\}_a = -\{\varepsilon_{abi}\varepsilon_{kji}A_bB_jC_k\}_a = -(\delta_{ak}\delta_{bj} - \delta_{aj}\delta_{bk})A_bB_jC_k$

$= -(A_jB_jC_k)_k + (A_kB_jC_k)_j = -(\vec{A}\cdot\vec{B})\vec{C} + (\vec{A}\cdot\vec{C})\vec{B}$ #

1.14. Cylindrical coordinate



$\vec{s} = r\hat{e}_r + z\hat{e}_z$,

$\hat{e}_r = (\cos\theta, \sin\theta, 0)$

$\hat{e}_\theta = (-\sin\theta, \cos\theta, 0)$

$\hat{e}_z = (0, 0, 1)$

$\vec{v} = \frac{d\vec{s}}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt} + \frac{dz}{dt}\hat{e}_z$

$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z$

$\frac{d\hat{e}_r}{dt} = (-\sin\theta\,\dot{\theta}, \cos\theta\,\dot{\theta}, 0) = \dot{\theta}\hat{e}_\theta$

$\frac{d\hat{e}_\theta}{dt} = (-\cos\theta\,\dot{\theta}, -\sin\theta\,\dot{\theta}, 0) = -\dot{\theta}\hat{e}_r$

$\vec{a} = \frac{d^2\vec{s}}{dt^2} = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta$
$+ r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}\dot{\theta}\hat{e}_r + \ddot{z}\hat{e}_z$

$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta + \ddot{z}\hat{e}_z$ #

1.16. Owooo.

gradient, div, curl.

$\nabla\phi$ $\quad \vec{\nabla}\cdot\vec{A}$ $\quad \nabla\times\vec{A}$

1.17.

$$\int_{surface} \vec{A}\cdot d\vec{a} = \int_{volume} \vec{\nabla}\cdot\vec{A}\, dv$$

$$\int_{path} \vec{A}\cdot d\vec{s} = \int_{surface} (\nabla\times\vec{A})\cdot d\vec{a}$$